8.6 Identify Special Quadrilaterals

|	
Before	You identified polygons.
Now	You will identify special quadrilaterals.
Why?	So you can describe part of a pyramid, as in Ex. 36.

Key Vocabulary
- **parallelogram,** *p. 515*
- **rhombus,** *p. 533*
- **rectangle,** *p. 533*
- **square,** *p. 533*
- **trapezoid,** *p. 542*
- **kite,** *p. 545*

The diagram below shows relationships among the special quadrilaterals you have studied in Chapter 8. Each shape in the diagram has the properties of the shapes linked above it. For example, a rhombus has the properties of a parallelogram and a quadrilateral.

Quadrilateral

Parallelogram   **Trapezoid**   **Kite**

Rectangle   **Rhombus**   **Isosceles trapezoid**
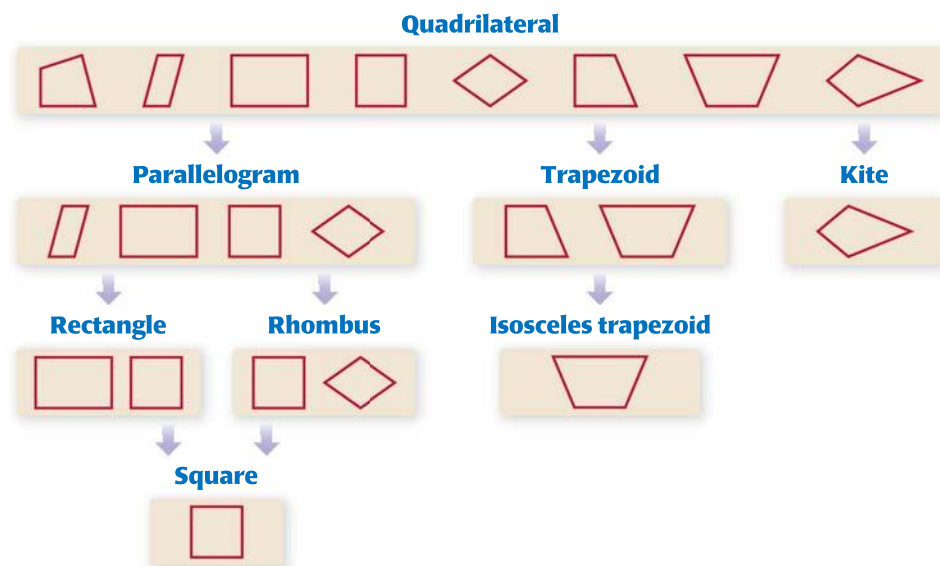
Square

EXAMPLE 1   **Identify quadrilaterals**

Quadrilateral *ABCD* has at least one pair of opposite angles congruent. What types of quadrilaterals meet this condition?

Solution

There are many possibilities.

Parallelogram	Rhombus	Rectangle	Square	Kite

Opposite angles are congruent.   All angles are congruent.   One pair of opposite angles are congruent.

What is the most specific name for quadrilateral *ABCD*?

Ⓐ Parallelogram   Ⓑ Rhombus
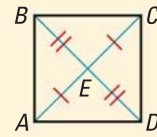
Ⓒ Square   Ⓓ Rectangle

Solution

The diagram shows $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$. So, the diagonals bisect each other. By Theorem 8.10, *ABCD* is a parallelogram.

Rectangles, rhombuses and squares are also parallelograms. However, there is no information given about the side lengths or angle measures of *ABCD*. So, you cannot determine whether it is a rectangle, a rhombus, or a square.

▸ The correct answer is A.   Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 3   **Identify a quadrilateral**

Is enough information given in the diagram to show that quadrilateral *PQRS* is an isosceles trapezoid? Explain.
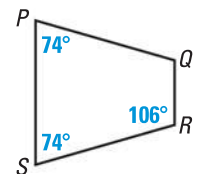
Solution

STEP 1 **Show** that *PQRS* is a trapezoid. ∠*R* and ∠*S* are supplementary, but ∠*P* and ∠*S* are not. So, $\overline{PS} \parallel \overline{QR}$, but $\overline{PQ}$ is not parallel to $\overline{SR}$. By definition, *PQRS* is a trapezoid.

STEP 2 **Show** that trapezoid *PQRS* is isosceles. ∠*P* and ∠*S* are a pair of congruent base angles. So, *PQRS* is an isosceles trapezoid by Theorem 8.15.

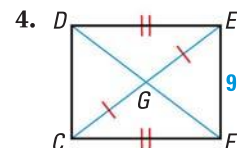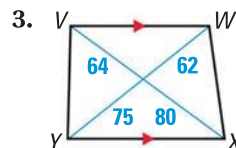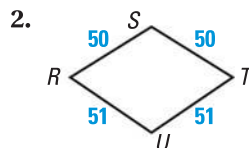▸ Yes, the diagram is sufficient to show that *PQRS* is an isosceles trapezoid.

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✓ **GUIDED PRACTICE**   **for Examples 1, 2, and 3**

1. Quadrilateral *DEFG* has at least one pair of opposite sides congruent. What types of quadrilaterals meet this condition?

Give the most specific name for the quadrilateral. *Explain* your reasoning.

2.   3.   4.

5. **ERROR ANALYSIS** A student knows the following information about quadrilateral *MNPQ*: $\overline{MN} \parallel \overline{PQ}$, $\overline{MP} \cong \overline{NQ}$, and ∠*P* ≅ ∠*Q*. The student concludes that *MNPQ* is an isosceles trapezoid. *Explain* why the student cannot make this conclusion.

8.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS**
on p. WS1 for Exs. 3, 15, and 33

★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 13, 37, and 38

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A quadrilateral that has exactly one pair of parallel sides and diagonals that are congruent is a(n) __?__.

2. ★ **WRITING** *Describe* three methods you could use to prove that a parallelogram is a rhombus.
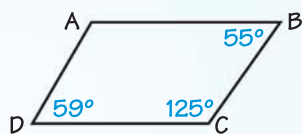
EXAMPLE 1
on p. 552
for Exs. 3–12

PROPERTIES OF QUADRILATERALS Copy the chart. Put an X in the box if the shape *always* has the given property.

Property	□	Rectangle	Rhombus	Square	Kite	Trapezoid
3. All sides are ≅.	?	?	?	?	?	?
4. Both pairs of opp. sides are ≅.	?	?	?	?	?	?
5. Both pairs of opp. sides are ∥.	?	?	?	?	?	?
6. Exactly 1 pair of opp. sides are ∥.	?	?	?	?	?	?
7. All ⦞ are ≅.	?	?	?	?	?	?
8. Exactly 1 pair of opp. ⦞ are ≅.	?	?	?	?	?	?
9. Diagonals are ⊥.	?	?	?	?	?	?
10. Diagonals are ≅.	?	?	?	?	?	?
11. Diagonals bisect each other.	?	?	?	?	?	?

12. **ERROR ANALYSIS** *Describe* and correct the error in classifying the quadrilateral.

∠B and ∠C are supplements, so $\overline{AB} \parallel \overline{CD}$. So, ABCD is a parallelogram.
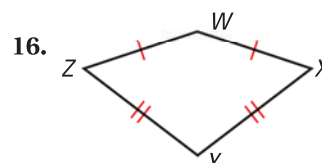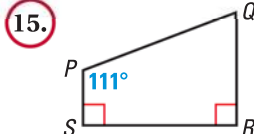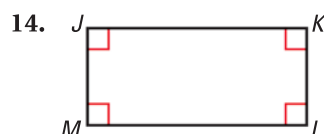
EXAMPLE 2
on p. 553
for Exs. 13–17

13. ★ **MULTIPLE CHOICE** What is the most specific name for the quadrilateral shown at the right?

Ⓐ Rectangle   Ⓑ Parallelogram

Ⓒ Trapezoid   Ⓓ Isosceles trapezoid

**CLASSIFYING QUADRILATERALS** Give the most specific name for the quadrilateral. *Explain.*
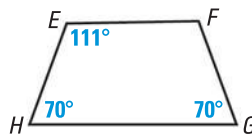
14.

15.

16.

17. DRAWING Draw a quadrilateral with congruent diagonals and exactly one pair of congruent sides. What is the most specific name for this quadrilateral?

IDENTIFYING QUADRILATERALS Tell whether enough information is given in the diagram to classify the quadrilateral by the indicated name. *Explain.*
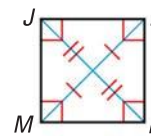
18. Rhombus
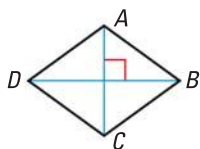
19. Isosceles trapezoid

20. Square

COORDINATE PLANE Points *P, Q, R,* and *S* are the vertices of a quadrilateral. Give the most specific name for *PQRS. Justify* your answer.

21. $P(1, 0)$, $Q(1, 2)$, $R(6, 5)$, $S(3, 0)$

22. $P(2, 1)$, $Q(6, 1)$, $R(5, 8)$, $S(3, 8)$

23. $P(2, 7)$, $Q(6, 9)$, $R(9, 3)$, $S(5, 1)$

**24.** $P(1, 7)$, $Q(5, 8)$, $R(6, 2)$, $S(2, 1)$

25. TECHNOLOGY Use geometry drawing software to draw points *A, B, C,* and segments *AC* and *BC*. Draw a circle with center *A* and radius *AC*. Draw a circle with center *B* and radius *BC*. Label the other intersection of the circles *D*. Draw $\overline{BD}$ and $\overline{AD}$.

a. Drag point *A, B, C,* or *D* to change the shape of *ABCD*. What types of quadrilaterals can be formed?

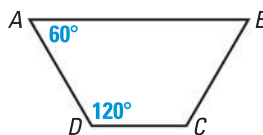**b.** Are there types of quadrilaterals that cannot be formed? *Explain.*
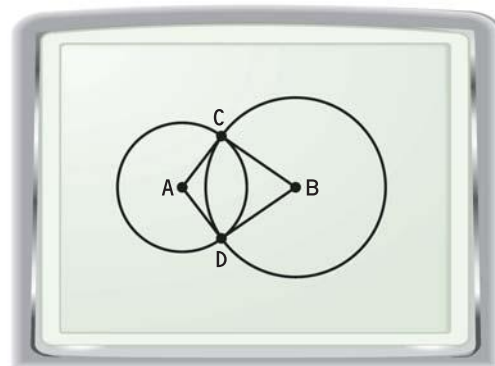
DEVELOPING PROOF Which pairs of segments or angles must be congruent so that you can prove that *ABCD* is the indicated quadrilateral? *Explain.* There may be more than one right answer.
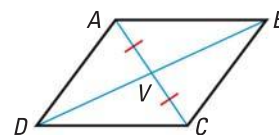
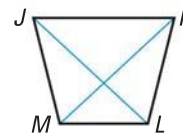**26.** Square
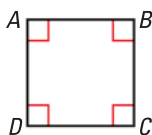
27. Isosceles trapezoid

28. Parallelogram

TRAPEZOIDS In Exercises 29–31, determine whether there is enough information to prove that *JKLM* is an isosceles trapezoid. *Explain.*

29. GIVEN ▶ $\overline{JK} \parallel \overline{LM}$, $\angle JKL \cong \angle KJM$

30. GIVEN ▶ $\overline{JK} \parallel \overline{LM}$, $\angle JML \cong \angle KLM$, $m\angle KLM \neq 90°$

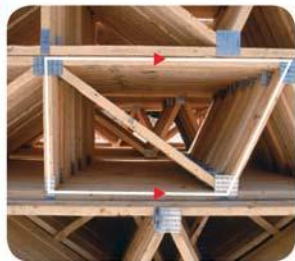**31. GIVEN** ▶ $\overline{JL} \cong \overline{KM}$, $\overline{JK} \parallel \overline{LM}$, $JK > LM$

32. CHALLENGE Draw a rectangle and bisect its angles. What type of quadrilateral is formed by the intersecting bisectors? *Justify* your answer.

REAL-WORLD OBJECTS What type of special quadrilateral is outlined?

33.

34.

35.

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36. PYRAMID Use the photo of the Pyramid of Kukulcan in Mexico.

  a. $\overline{EF} \parallel \overline{HG}$, and $\overline{EH}$ and $\overline{FG}$ are not parallel. What shape is this part of the pyramid?

  b. $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$, and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are all congruent to each other. What shape is this part of the pyramid?


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37. ★ SHORT RESPONSE *Explain* why a parallelogram with one right angle must be a rectangle.

38. ★ EXTENDED RESPONSE Segments *AC* and *BD* bisect each other.

  a. Suppose that $\overline{AC}$ and $\overline{BD}$ are congruent, but not perpendicular. Draw quadrilateral *ABCD* and classify it. *Justify* your answer.

  b. Suppose that $\overline{AC}$ and $\overline{BD}$ are perpendicular, but not congruent. Draw quadrilateral *ABCD* and classify it. *Justify* your answer.

39. MULTI-STEP PROBLEM Polygon *QRSTUV* shown at the right is a regular hexagon, and $\overline{QU}$ and $\overline{RT}$ are diagonals. Follow the steps below to classify quadrilateral *QRTU*. *Explain* your reasoning in each step.
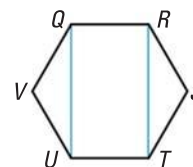

  a. Show that $\triangle QVU$ and $\triangle RST$ are congruent isosceles triangles.

  b. Show that $\overline{QR} \cong \overline{UT}$ and that $\overline{QU} \cong \overline{RT}$.

  c. Show that $\angle UQR \cong \angle QRT \cong \angle RTU \cong \angle TUQ$. Find the measure of each of these angles.

  d. Classify quadrilateral *QRTU*.

40. REASONING In quadrilateral *WXYZ*, $\overline{WY}$ and $\overline{XZ}$ intersect each other at point *V*. $\overline{WV} \cong \overline{XV}$ and $\overline{YV} \cong \overline{ZV}$, but $\overline{WY}$ and $\overline{XZ}$ do not bisect each other. Draw $\overline{WY}$, $\overline{XY}$, and *WXYZ*. What special type of quadrilateral is *WXYZ*? Write a plan for a proof of your answer.

○ **= WORKED-OUT SOLUTIONS**
  on p. WS1

★ **= STANDARDIZED**
  TEST PRACTICE

CHALLENGE What special type of quadrilateral is *EFGH*? Write a paragraph proof to show that your answer is correct.
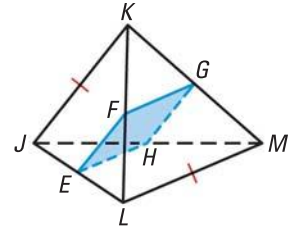
41. **GIVEN** ▶ *PQRS* is a square.
E, F, G, and H are midpoints of the sides of the square.

PROVE ▶ *EFGH* is a __?__.
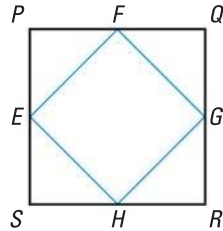
42. **GIVEN** ▶ In the three-dimensional figure, $\overline{JK} \cong \overline{LM}$; E, F, G, and H are the midpoints of $\overline{JL}$, $\overline{KL}$, $\overline{KM}$, and $\overline{JM}$.
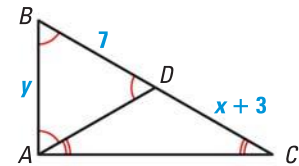
PROVE ▶ *EFGH* is a __?__.

MIXED REVIEW

In Exercises 43 and 44, use the diagram. *(p. 264)*

43. Find the values of *x* and *y*. *Explain* your reasoning.

44. Find *m∠ADC*, *m∠DAC*, and *m∠DCA*. *Explain* your reasoning.

The vertices of quadrilateral *ABCD* are *A*(−2, 1), *B*(2, 5), *C*(3, 2), and *D*(1, −1). Draw *ABCD* in a coordinate plane. Then draw its image after the indicated translation. *(p. 272)*

45. $(x, y) \rightarrow (x + 1, y - 3)$ **46.** $(x, y) \rightarrow (x - 2, y - 2)$
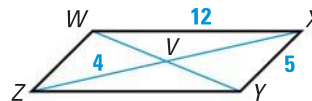
Use the diagram of ▱WXYZ to find the indicated length. *(p. 515)*
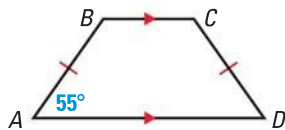
47. *YZ*   **48.** *WZ*

49. *XV*   **50.** *XZ*

QUIZ *for Lessons 8.5–8.6*

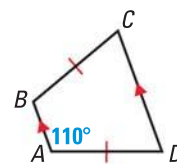**Find the unknown angle measures.** *(p. 542)*

1.

2.
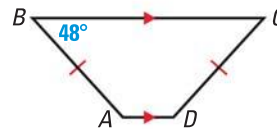
3.

4. The diagonals of quadrilateral *ABCD* are congruent and bisect each other. What types of quadrilaterals match this description? *(p. 552)*

5. In quadrilateral *EFGH*, ∠*E* ≅ ∠*G*, ∠*F* ≅ ∠*H*, and $\overline{EF} \cong \overline{EH}$. What is the most specific name for quadrilateral *EFGH*? *(p. 552)*